

UWThPh-1995-14

June 6, 1995

The decay $\rho^0 \rightarrow \pi^+\pi^-\gamma$ and the chiral invariant interactions of vector mesons*

K. Huber and H. Neufeld

Institut für Theoretische Physik der Universität Wien
Boltzmanngasse 5, A-1090 Wien, Austria

Abstract

Using the close relationship between the low-energy constants of chiral perturbation theory and the chiral invariant interactions of the vector meson resonances with the pseudoscalar mesons, we investigate the process $\rho^0 \rightarrow \pi^+\pi^-\gamma$. Compared with the contribution from the pure bremsstrahlung mechanism, we find an enhancement of the decay rate near the endpoint of the photon energy spectrum. Such a particular shape of the differential decay rate has indeed been observed experimentally and turns out to be an important confirmation of the theoretical concept of chiral vector meson dominance.

* Work supported in part by Fonds zur Förderung der wissenschaftlichen Forschung (Austria), Project No. P09505-PHY and by HCM, EEC-Contract No. CHRX-CT920026 (EURODAΦNE).

1 Introduction

At low energies, the interactions of the pseudoscalar mesons are described by chiral perturbation theory (χ PT) [1, 2, 3]. This effective low-energy theory is mathematically equivalent [4] to QCD, the underlying fundamental theory. The lagrangian of χ PT contains all terms compatible with the symmetries of QCD. The associated coupling constants can be interpreted as describing the influence of all degrees of freedom not explicitly contained in the chiral lagrangian.

It has been shown [5, 6, 7] that the coupling constants which occur at $\mathcal{O}(p^4)$ in the low-energy expansion are dominated by the low-lying vector, axial-vector, scalar and pseudoscalar resonances. In particular, chiral symmetry was found to confirm the phenomenologically successful concept of vector meson dominance [8].

In this letter we shall employ this connexion between the chiral invariant vector meson interactions and the low-energy constants of χ PT. It is possible to obtain rather precise results for the decay spectrum of $\rho^0 \rightarrow \pi^+\pi^-\gamma$ which can be confronted with the present experimental data [9, 10].

2 The low-energy limit of QCD

The strong, electromagnetic and semileptonic interactions of pseudoscalar mesons are described by an effective chiral lagrangian \mathcal{L}_{eff} . It consists of a string of terms

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \dots, \quad (2.1)$$

organized in powers of momenta and meson masses, respectively. The lowest order term \mathcal{L}_2 is the nonlinear sigma model lagrangian in the presence of external fields¹:

$$\mathcal{L}_2 = \frac{F^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle, \quad (2.2)$$

The generating functional $Z[v, a, s, p]$ for the Green functions of vector, axial-vector, scalar and pseudoscalar quark currents is given by the expansion of the effective meson field theory in the number of loops,

$$Z = Z_2 + Z_4 + Z_6 + \dots. \quad (2.3)$$

The leading term coincides with the classical action associated with \mathcal{L}_2 .

At next to leading order p^4 the generating functional consists of the following terms: one-loop graphs generated by the vertices of \mathcal{L}_2 , tree graphs involving one vertex from \mathcal{L}_4 and finally a contribution Z_{WZW} to account for the chiral anomaly. The chiral invariant lagrangian \mathcal{L}_4 of $\mathcal{O}(p^4)$ is given by [3]

$$\begin{aligned} \mathcal{L}_4 = & L_1 \langle u_\mu u^\mu \rangle^2 + L_2 \langle u_\mu u^\nu \rangle \langle u^\mu u_\nu \rangle + L_3 \langle u_\mu u^\mu u_\nu u^\nu \rangle + L_4 \langle u_\mu u^\mu \rangle \langle \chi_+ \rangle \\ & + L_5 \langle u_\mu u^\mu \chi_+ \rangle + L_6 \langle \chi_+ \rangle^2 + L_7 \langle \chi_- \rangle^2 + \frac{1}{4}(2L_8 + L_{12}) \langle \chi_+^2 \rangle + \frac{1}{4}(2L_8 - L_{12}) \langle \chi_-^2 \rangle \\ & - iL_9 \langle f_+^{\mu\nu} u_\mu u_\nu \rangle + \frac{1}{4}(L_{10} + 2L_{11}) \langle f_{+\mu\nu} f_+^{\mu\nu} \rangle - \frac{1}{4}(L_{10} - 2L_{11}) \langle f_{-\mu\nu} f_-^{\mu\nu} \rangle. \end{aligned} \quad (2.4)$$

¹Our notation is the same as in refs. [5, 6].

The twelve low-energy couplings L_1, \dots, L_{12} arising here are divergent (except L_3, L_7). They absorb the divergences of the one-loop graphs via the renormalization

$$\begin{aligned} L_i &= L_i^r(\mu) + \Gamma_i \Lambda(\mu), \\ \Lambda(\mu) &= \frac{\mu^{d-4}}{(4\pi)^2} \left\{ \frac{1}{d-4} - \frac{1}{2} [\ln(4\pi) + \Gamma'(1) + 1] \right\}, \end{aligned} \quad (2.5)$$

in the dimensional regularization scheme.

The meson decay constant $F \simeq F_\pi = 92.4$ MeV [11], the vacuum condensate parameter B_0 (contained in χ_+), together with L_1^r, \dots, L_{10}^r , determine the low-energy behaviour of pseudoscalar meson interactions to $\mathcal{O}(p^4)$. L_{11}^r and L_{12}^r are contact terms which are not directly accessible to experiment. The coupling constants L_1^r, \dots, L_{10}^r have been determined from experimental input and by using large- N_c arguments [3]. An updated list of their numerical values can be found in ref. [12].

3 Chiral couplings of vector resonances

To investigate the contribution of vector meson exchange to the L_i^r , one has to include spin-1 fields in \mathcal{L}_{eff} . Following ref. [5], we describe the vector meson octet (ρ, K^*, ω_8) in terms of a 3×3 matrix valued antisymmetric tensor field $V_{\mu\nu}$. The kinetic term of the vector meson lagrangian takes the form

$$\mathcal{L}_{\text{kin}}(V) = -\frac{1}{2} \left\langle \nabla^\lambda V_{\lambda\mu} \nabla_\nu V^{\nu\mu} - \frac{M_V^2}{2} V_{\mu\nu} V^{\mu\nu} \right\rangle. \quad (3.1)$$

To lowest order p^2 in the chiral expansion, the most general interaction of a single vector meson with the pseudoscalar bosons (π, K, η) is described by the lagrangian [5]

$$\mathcal{L}_2(V) = \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{iG_V}{\sqrt{2}} \langle V_{\mu\nu} u^\mu u^\nu \rangle, \quad (3.2)$$

where the two real coupling constants F_V and G_V are not further restricted by chiral symmetry. Their numerical values can be determined from the decay rates for $\rho^0 \rightarrow e^+e^-$ and $\rho \rightarrow \pi\pi$. From (3.2) one obtains

$$\Gamma(\rho^0 \rightarrow e^+e^-) = \frac{4\pi\alpha^2 F_V^2}{3M_\rho} \left(1 + \frac{2m_e^2}{M_\rho^2}\right) \left(1 - \frac{4m_e^2}{M_\rho^2}\right)^{1/2}, \quad (3.3)$$

and

$$\Gamma(\rho \rightarrow \pi\pi) = \frac{G_V^2 M_\rho^3}{48\pi F_\pi^4} \left(1 - \frac{4M_\pi^2}{M_\rho^2}\right)^{3/2}, \quad (3.4)$$

respectively. Comparison with the experimental value $\Gamma(\rho^0 \rightarrow e^+e^-) = (6.7 \pm 0.3)$ keV [11] yields

$$|F_V| \simeq 153 \text{ MeV}, \quad (3.5)$$

while $\Gamma(\rho \rightarrow \pi\pi) = (151.2 \pm 1.2)$ MeV [11] implies

$$|G_V| \simeq 67 \text{ MeV}. \quad (3.6)$$

Higher order chiral corrections may reduce this value to $|G_V| \simeq 53$ MeV [5].

To order p^4 in the chiral expansion, vector meson exchange induces a local lagrangian of the type (2.4) with [5]

$$\begin{aligned} L_1^V &= \frac{G_V^2}{8M_V^2}, & L_2^V &= 2L_1^V, & L_3^V &= -6L_1^V, \\ L_9^V &= \frac{F_V G_V}{2M_V^2}, & L_{10}^V &= -\frac{F_V^2}{4M_V^2}, \\ L_4^V &= L_5^V = L_6^V = L_7^V = L_8^V = 0. \end{aligned} \quad (3.7)$$

With $M_V = M_\rho$, the information on $|F_V|$ from $\rho^0 \rightarrow e^+e^-$ and on $|G_V|$ from $\rho \rightarrow \pi\pi$, the contributions (3.7) (including L_{10}^A from axial–vector exchange) were found [5] to saturate the phenomenological values of the renormalized coupling constants $L_i^r(M_\rho)$ for $i = 1, 2, 3, 9, 10$. In addition, the coupling constants with vanishing contributions from the vector (or axial–vector) mesons are saturated by scalar and pseudoscalar exchange, respectively [5]. These findings can be summarized by means of the following catch–words [13]:

- **Chiral duality:** The $L_i^r(M_\rho)$ are practically saturated by resonance exchange.
- **Chiral vector meson dominance:** Whenever spin–1 resonances can contribute at all ($i = 1, 2, 3, 9, 10$), the $L_i^r(M_\rho)$ are almost completely dominated by vector and axial–vector exchange.

Furthermore, with additional QCD–inspired assumptions, the vector and axial–vector contributions to the L_i can be expressed in terms of F_π and $M_V \simeq M_\rho$ [6],

$$8L_1^V = 4L_2^V = -\frac{4}{3}L_3^V = L_9^V = -\frac{4}{3}L_{10}^{V+A} = \frac{F_\pi^2}{2M_V^2}, \quad (3.8)$$

in very good agreement with the phenomenological values of the $L_i^r(M_\rho)$.

4 The decay $\rho^0 \rightarrow \pi^+\pi^-\gamma$

The observed rates for $\rho^0 \rightarrow e^+e^-$ and $\rho \rightarrow \pi\pi$ permit only the determination of the absolute values of the coupling constants F_V, G_V . In the case of $\rho^0 \rightarrow \pi^+\pi^-\gamma$ there is also a contribution from an interference term between F_V and G_V which provides us with the additional experimental information about the sign of the product $F_V G_V$. As we have seen in the last section, chiral vector meson dominance relates this quantity to the low–energy constant L_9^r :

$$L_9^r(M_\rho) \simeq L_9^V = \frac{F_V G_V}{2M_V^2}. \quad (4.1)$$

On the other hand, a very precise numerical value [12] for $L_9^r(M_\rho)$ has been extracted from the pion charge radius:

$$L_9^r(M_\rho) = (6.9 \pm 0.7) \cdot 10^{-3}. \quad (4.2)$$

Together with (4.1), this clearly implies $F_V G_V > 0$. The same conclusion can, of course, also be drawn from (3.8).

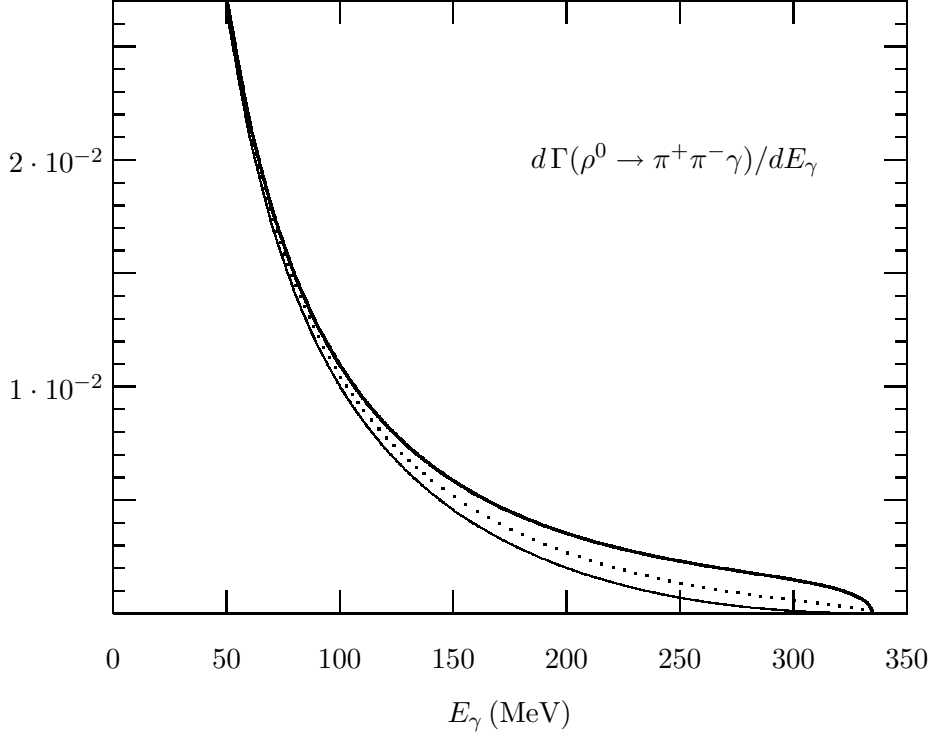


Figure 1: Photon spectrum of $\rho^0 \rightarrow \pi^+\pi^-\gamma$ for $F_V G_V > 0$ (thick solid curve), $F_V = 0$ (dotted curve) and $F_V G_V < 0$ (thin solid curve).

Taking the relevant vertices from (3.2) and (2.2), the decay rate of $\rho^0 \rightarrow \pi^+\pi^-\gamma$ can easily be calculated². The photon spectrum is given by

$$\begin{aligned} \frac{d\Gamma}{dE_\gamma} = & \frac{e^2 M_\rho^2}{96\pi^3 F_\pi^4 z} \left\{ \left[F_V^2 z^4 + 2F_V G_V z^2 (1 - 2z^2) + G_V^2 (4z^4 + (2z - 1)(1 - 4r_\pi^2)) \right] w(z) \right. \\ & \left. + \left[-4F_V G_V r_\pi^2 z^2 + G_V^2 (1 - 2z + 2r_\pi^2 (4z^2 + 4z - 3 + 4r_\pi^2)) \right] \ln \frac{1 + w(z)}{1 - w(z)} \right\}, \quad (4.3) \end{aligned}$$

with

$$z = E_\gamma/M_\rho, \quad w(z) = \sqrt{1 - \frac{4r_\pi^2}{1 - 2z}}, \quad r_\pi = M_\pi/M_\rho, \quad (4.4)$$

where E_γ is the photon energy in the rest frame of the ρ^0 .

In our numerical analysis, the input parameters F_V, G_V have been chosen in such a way that the central values of the present experimental data for $\Gamma(\rho^0 \rightarrow e^+e^-)$ and $\Gamma(\rho \rightarrow \pi\pi)$ (see sect. 3) are reproduced by (3.3) and (3.4), respectively. Our results for $\rho^0 \rightarrow \pi^+\pi^-\gamma$ are displayed in fig. 1. The thick solid curve shows the theoretically expected differential decay rate with $F_V G_V > 0$. For comparison, the dotted line gives the contribution of pure bremsstrahlung ($F_V = 0$), while the thin solid curve corresponds to $F_V G_V < 0$.

²An older theoretical treatment can be found in Ref. [14].

For small photon energies, the decay rate is, of course, dominated by bremsstrahlung. However, at the upper end of the photon spectrum, the solution favoured by chiral vector meson dominance shows a sizable enhancement compared to bremsstrahlung alone.

Experimentally, the decay rate of $\rho^0 \rightarrow \pi^+\pi^-\gamma$ has been determined at the e^+e^- storage ring VEPP-2M at Novosibirsk [9, 10]. Fig. 16 of ref. [10] shows the observed photon spectrum in the energy range $E_\gamma > 50$ MeV. The measured values are rather close to the rate expected from pure bremsstrahlung, except for the last bin ($E_\gamma > 300$ MeV) where a significant enhancement (three standard deviations) has been observed.

These experimental results are in perfect agreement with our theoretical expectations. Not only the presence of the F_V coupling in $\rho^0 \rightarrow \pi^+\pi^-\gamma$ has been detected but also the positive sign of $F_V G_V$ has been clearly established. Note that $F_V G_V < 0$ would have implied a practically vanishing contribution to the decay rate for $E_\gamma > 300$ MeV.

Let us finally compare the theoretical and the experimental branching ratio. For $F_V G_V > 0$ we find

$$BR(\rho^0 \rightarrow \pi^+\pi^-\gamma) = 1.1 \cdot 10^{-2} \quad \text{for } E_\gamma > 50 \text{ MeV}, \quad (4.5)$$

while the measured value is given by [10]

$$BR(\rho^0 \rightarrow \pi^+\pi^-\gamma) = (0.99 \pm 0.04 \pm 0.15) \cdot 10^{-2} \quad \text{for } E_\gamma > 50 \text{ MeV}. \quad (4.6)$$

5 Summary

The present experimental data for the decay $\rho^0 \rightarrow \pi^+\pi^-\gamma$ can be consistently described by the lowest-order vector meson lagrangian (3.2) with two coupling constants F_V, G_V . The observed shape of the photon spectrum unambiguously implies a positive sign for the product $F_V G_V$. As this quantity can be related to the low-energy constant $L_9^r(M_\rho)$ of χ PT, the experimental results constitute an important test of the notion of chiral vector meson dominance.

Acknowledgements

We wish to thank G. Ecker for useful discussions.

References

- [1] S. Weinberg, *Physica A* 96 (1979) 327.
- [2] J. Gasser and H. Leutwyler, *Ann. Phys.* 158 (1984) 142.
- [3] J. Gasser and H. Leutwyler, *Nucl. Phys. B* 250 (1985) 465.
- [4] H. Leutwyler, *Ann. Phys.* 235 (1994) 165.
- [5] G. Ecker, J. Gasser, A. Pich and E. de Rafael, *Nucl. Phys. B* 321 (1989) 311.
- [6] G. Ecker, J. Gasser, H. Leutwyler, A. Pich and E. de Rafael, *Phys. Lett. B* 223 (1989) 425.
- [7] J.F. Donoghue, C. Ramirez and G. Valencia, *Phys. Rev. D* 39 (1989) 1947.
- [8] J.J. Sakurai, *Currents and mesons* (University of Chicago Press, Chicago, 1969).
- [9] I.B. Vasserman et al., *Yad. Fiz.* 47 (1988) 1635 [*Sov. J. Nucl. Phys.* 47 (1988) 1035].
- [10] S.I. Dolinsky et al., *Phys. Rep.* 202 (1991) 99.
- [11] Particle Data Group, L. Montanet et al., *Phys. Rev. D* 50 (1994), Number 3, Part I.
- [12] J. Bijnens, G. Ecker and J. Gasser, in: *The second DAΦNE physics handbook*, eds. L. Maiani, G. Pancheri, N. Paver (INFN, Frascati, 1995).
- [13] G. Ecker, Estimates of low-energy parameters, Univ. Wien preprint UWThPh-1994-34, to appear in: *Proc. of the workshop on chiral dynamics: theory and experiment* (MIT, Cambridge, July 1994).
- [14] P. Singer, *Phys. Rev.* 130 (1963) 2441.